AIM HIGH, TOGETHER WE FLY

THE VILLAGE
INTERNATIONAL SCHOOL
"We Nurture Dreams"

Q.No.	ANSWERS
	SECTION A
1	(C) 31
2	(A)2
3	(A) 3KM
4	(B)[-7,3]
5	(C)
6	32/3
7	(A) RS 3,000
8	(C) Rs 40,600
9	(B) Rs 10,000
10	(B) Compound Annual Growth Rate
11.	(C) Irregular Trend
12.	(D) All the above
13	(D) any point on the line segment joining the points (0, 2) and (3, 0).
14	(A) m = np
15.	(a) 0
16.	(A) Bell Shaped
17.	(D) III only
18	(D) a parameter
19	(a)
20	(c)
	SECTION B
21	Speed of still water (x) = 8 km/hr Speed of the stream (y) = 4 km/hr Distance Covered (d) = 16 km Speed of upstream (v) = x – y = 8 – 4 = 4 km/hr Time Taken in upstream = d/s = 16/4 = 4 hour
22	$\begin{bmatrix} 5 & 3x \\ 2y & z \end{bmatrix} = \begin{bmatrix} 5 & 12 \\ 6 & 4 \end{bmatrix}$ $\therefore 3x = 12 \Rightarrow x = 4 \ and \ 2y = 6 \Longrightarrow y = 3 \ and \ z = 4$

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i = $\frac{0.06}{2}$= 0.03

We know that

P = $\frac{R}{i}$= 40,000

40000 =$\frac{x}{0.03}$

x = R.S 1200

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Let sample space S = { 1,2,3,4,5,6 }

Probability distribution :

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

OR

Mean $=\sum x_i P(x_i) ==\left(0 \times \frac{1}{10} + 1 \times \frac{1}{2} + 2 \times \frac{2}{5}\right)$

= ½ + 4/5 = $\frac{(5+8)}{10}$ = 13/10 =1.3

25

H0 : µ = 0.50 mm and H1 : µ ǂ0.50 mm

Thus two tailed test is applied under hypothesis H0

$$t = \frac{\bar{X}-\mu}{s} \times \sqrt{n-1} = \frac{0.53-0.50}{0.03} \times \sqrt{9} = 3$$

Since t(=3) > $t_{0.025}$(2.262), the null hypothesis H_0 can be rejected. Hence we conclude that machine is not working properly.

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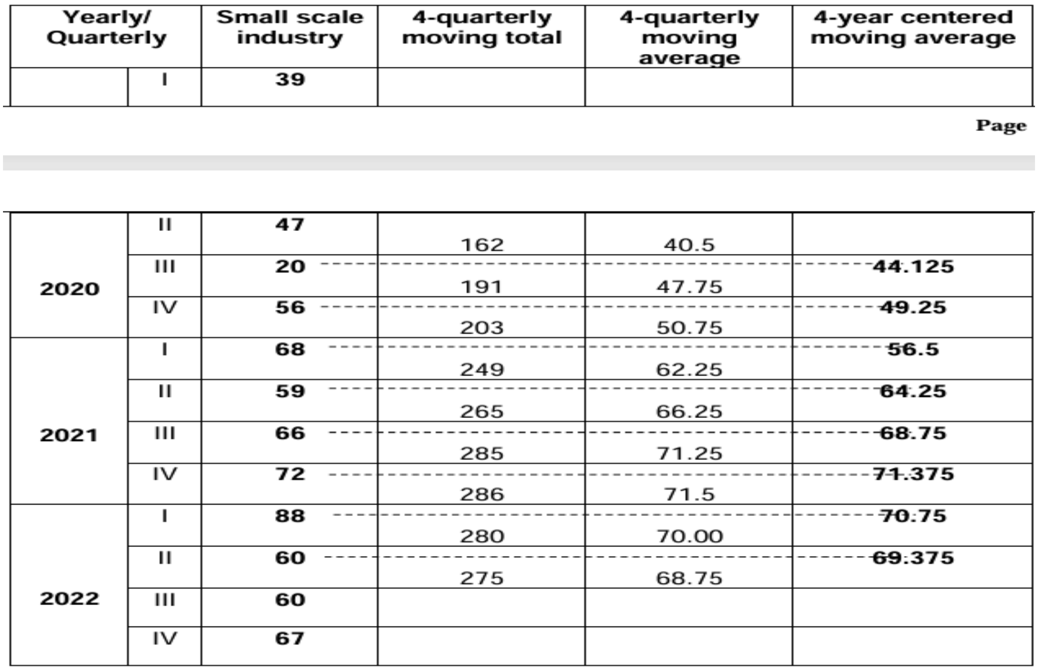

Yearly/ Quarterly		Small scale industry	4-quarterly moving total	4-quarterly moving average	4-year centered moving average
	I	39			
2020	II	47			
			162	40.5	
	III	20			44.125
			191	47.75	
	IV	56			49.25
			203	50.75	
2021	I	68			56.5
			249	62.25	
	II	59			64.25
			265	66.25	
	III	66			68.75
			285	71.25	
	IV	72			71.375
			286	71.5	
2022	I	88			70.75
			280	70.00	
	II	60			69.375
			275	68.75	
	III	60			
	IV	67			

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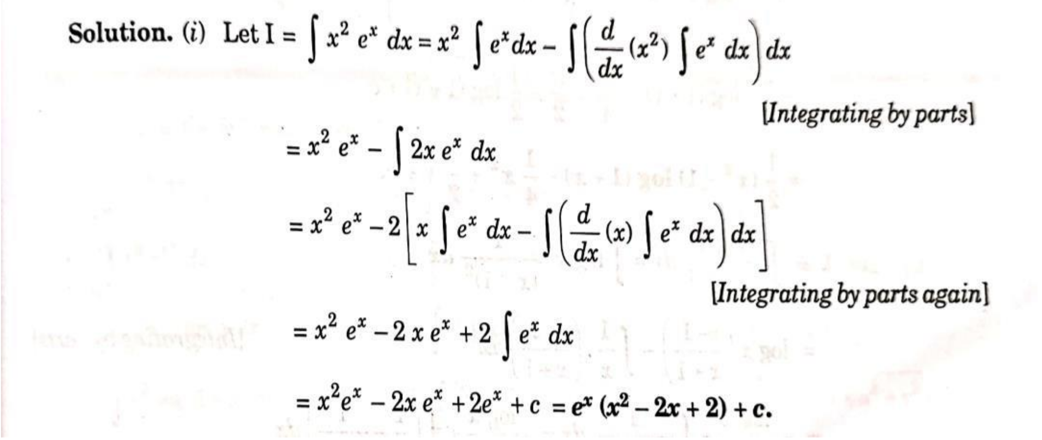


Solution. (*i*) Let $I = \int x^2 e^x\,dx = x^2 \int e^x dx - \int\left(\frac{d}{dx}(x^2)\int e^x\,dx\right)dx$

[*Integrating by parts*]

$$= x^2 e^x - \int 2x\,e^x\,dx$$

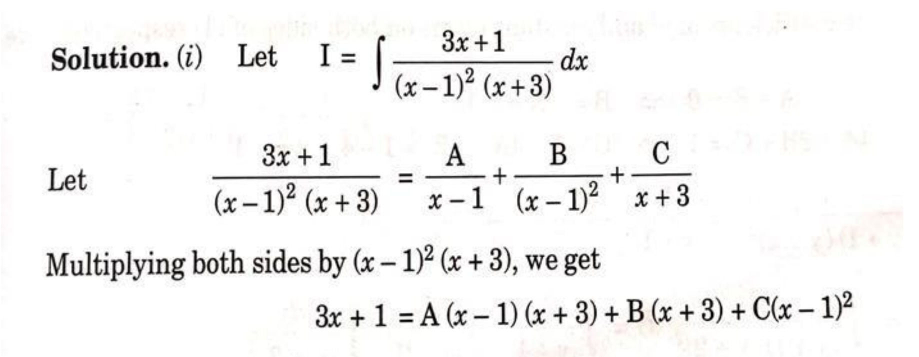
$$= x^2 e^x - 2\left[x\int e^x\,dx - \int\left(\frac{d}{dx}(x)\int e^x\,dx\right)dx\right]$$

[*Integrating by parts again*]

$$= x^2 e^x - 2\,x\,e^x + 2\int e^x\,dx$$

$$= x^2e^x - 2x\,e^x + 2e^x + c = \mathbf{e^x\,(x^2 - 2x + 2) + c.}$$

OR



Solution. (*i*) Let $I = \int \frac{3x+1}{(x-1)^2\,(x+3)}\,dx$

Let $\frac{3x+1}{(x-1)^2\,(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}$

Multiplying both sides by $(x-1)^2\,(x+3)$, we get

$$3x + 1 = A\,(x-1)\,(x+3) + B\,(x+3) + C(x-1)^2$$

Putting $x = 1$ in (2), we get : $4 = B\,.\,(4) \quad \Rightarrow \quad B = 1$

Putting $x = -3$ in (2), we get : $-8 = C\,.\,(16) \quad \Rightarrow \quad C = -\dfrac{1}{2}$

Comparing the coefficients of x^2 on both sides of (2), we get

$$0 = A + C \quad \Rightarrow \quad A = -C = \frac{1}{2}$$

$$\therefore \quad \frac{3x+1}{(x-1)^2\,(x+3)} = \frac{1}{2\,(x-1)} + \frac{1}{(x-1)^2} - \frac{1}{2(x+3)}$$

$$\therefore \quad I = \int \left(\frac{1}{2(x-1)} + \frac{1}{(x-1)^2} - \frac{1}{2(x+3)} \right) dx$$

$$= \frac{1}{2} \int \frac{1}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx - \frac{1}{2} \int \frac{1}{(x+3)}\,dx$$

$$= \boldsymbol{\frac{1}{2} \log|x-1| - \frac{1}{x-1} - \frac{1}{2} \log|x+3| + c.}$$

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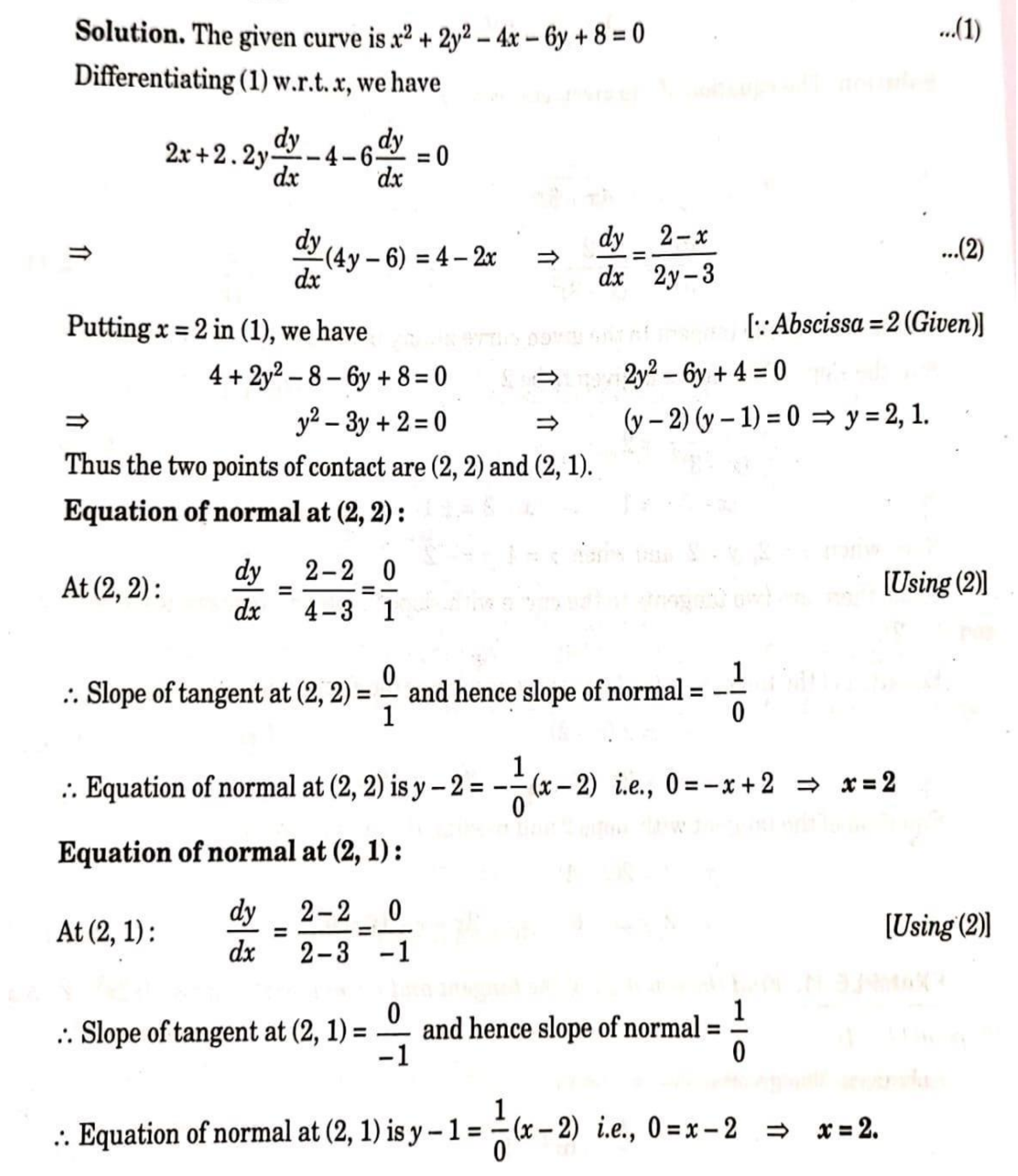


Solution. The given curve is $x^2 + 2y^2 - 4x - 6y + 8 = 0$ ...(1)

Differentiating (1) w.r.t. x, we have

$$2x + 2 \,.\, 2y\frac{dy}{dx} - 4 - 6\frac{dy}{dx} = 0$$

$$\Rightarrow \qquad \frac{dy}{dx}(4y-6) = 4 - 2x \quad \Rightarrow \quad \frac{dy}{dx} = \frac{2-x}{2y-3} \qquad ...(2)$$

Putting $x = 2$ in (1), we have [$\because$ *Abscissa = 2 (Given)*]

$$4 + 2y^2 - 8 - 6y + 8 = 0 \quad \Rightarrow \quad 2y^2 - 6y + 4 = 0$$

$$\Rightarrow \qquad y^2 - 3y + 2 = 0 \quad \Rightarrow \quad (y-2)(y-1) = 0 \Rightarrow y = 2, 1.$$

Thus the two points of contact are (2, 2) and (2, 1).

Equation of normal at (2, 2) :

$$\text{At } (2, 2): \quad \frac{dy}{dx} = \frac{2-2}{4-3} = \frac{0}{1} \qquad [\textit{Using } (2)]$$

$\therefore$ Slope of tangent at (2, 2) = $\frac{0}{1}$ and hence slope of normal = $-\frac{1}{0}$

$\therefore$ Equation of normal at (2, 2) is $y - 2 = -\frac{1}{0}(x-2)$ *i.e.,* $0 = -x + 2 \Rightarrow \boldsymbol{x = 2}$

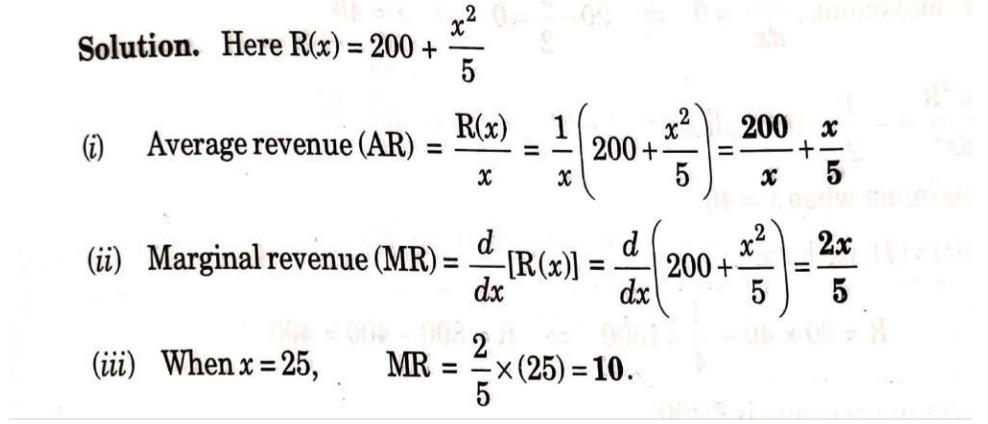
Equation of normal at (2, 1) :

$$\text{At } (2, 1): \quad \frac{dy}{dx} = \frac{2-2}{2-3} = \frac{0}{-1} \qquad [\textit{Using } (2)]$$

$\therefore$ Slope of tangent at (2, 1) = $\frac{0}{-1}$ and hence slope of normal = $\frac{1}{0}$

$\therefore$ Equation of normal at (2, 1) is $y - 1 = \frac{1}{0}(x-2)$ *i.e.,* $0 = x - 2 \Rightarrow \boldsymbol{x = 2.}$

OR

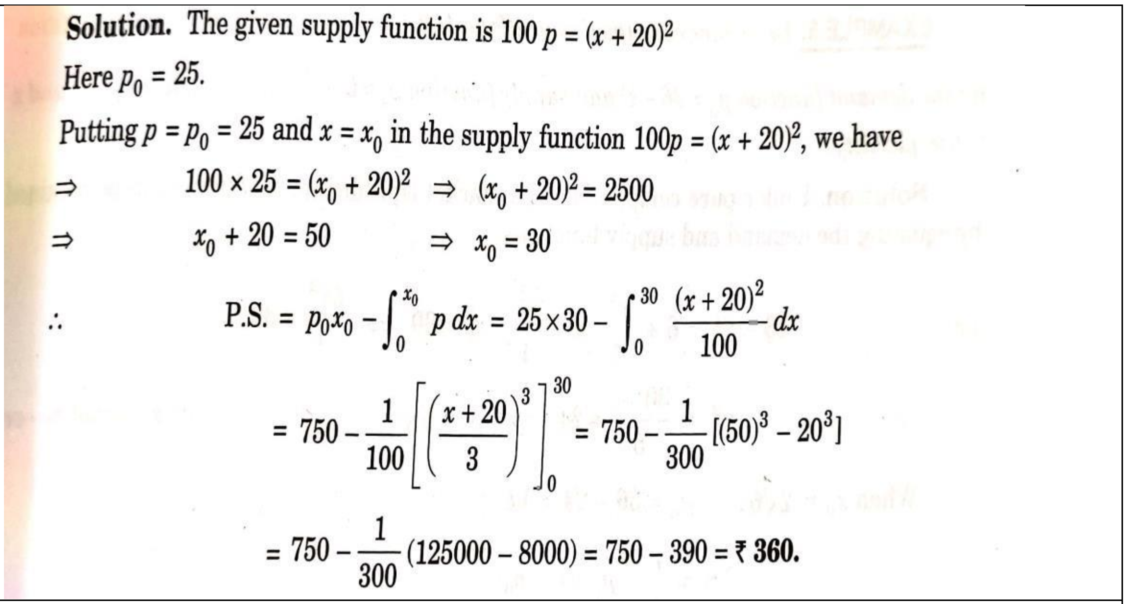


Solution. Here $R(x) = 200 + \frac{x^2}{5}$

(*i*) Average revenue (AR) = $\frac{R(x)}{x} = \frac{1}{x}\left(200 + \frac{x^2}{5}\right) = \boldsymbol{\frac{200}{x} + \frac{x}{5}}$

(*ii*) Marginal revenue (MR) = $\frac{d}{dx}[R(x)] = \frac{d}{dx}\left(200 + \frac{x^2}{5}\right) = \boldsymbol{\frac{2x}{5}}$

(*iii*) When $x = 25$, MR = $\frac{2}{5} \times (25) = \mathbf{10}$.

29	 **Solution.** The given supply function is $100\,p = (x + 20)^2$ Here $p_0 = 25$. Putting $p = p_0 = 25$ and $x = x_0$ in the supply function $100p = (x + 20)^2$, we have $\Rightarrow \quad 100 \times 25 = (x_0 + 20)^2 \quad \Rightarrow \quad (x_0 + 20)^2 = 2500$ $\Rightarrow \quad x_0 + 20 = 50 \quad \Rightarrow \quad x_0 = 30$ $\therefore \quad \text{P.S.} = p_0x_0 - \int_0^{x_0} p\,dx = 25 \times 30 - \int_0^{30} \frac{(x+20)^2}{100}\,dx$ $= 750 - \frac{1}{100}\left[\left(\frac{x+20}{3}\right)^3\right]_0^{30} = 750 - \frac{1}{300}[(50)^3 - 20^3]$ $= 750 - \frac{1}{300}(125000 - 8000) = 750 - 390 =$ ₹ **360.**
30	$\text{I} = \frac{10}{100}(200000)(5) = 100000$ n = 5 years = 5x12 = 60 EMI is given by the formula $EMI = \frac{P+I}{n}$ $EMI = \frac{200000+100000}{60}$ = R.S 5000
31	Standard deviation= npq=8 Mean = np=10 By dividing npq/np = 8/10 q= 4/5 p=1-q =1-4/5 =1/5 np= 10 n=50 p(X=r)= $50_{C_r}\left(\frac{1}{5}\right)^r\left(\frac{4}{5}\right)^{50-r}$ OR Mean = n Variance= npq Sum= np+npq n=5 5p+5pq=4.8 10p-5p2=4.8 p=1.2, 0.8 p=0.8=4/5 q=1-p 1/5 p(X=r)= $5_{C_r}\left(\frac{4}{5}\right)^r\left(\frac{1}{5}\right)^{5-r}$

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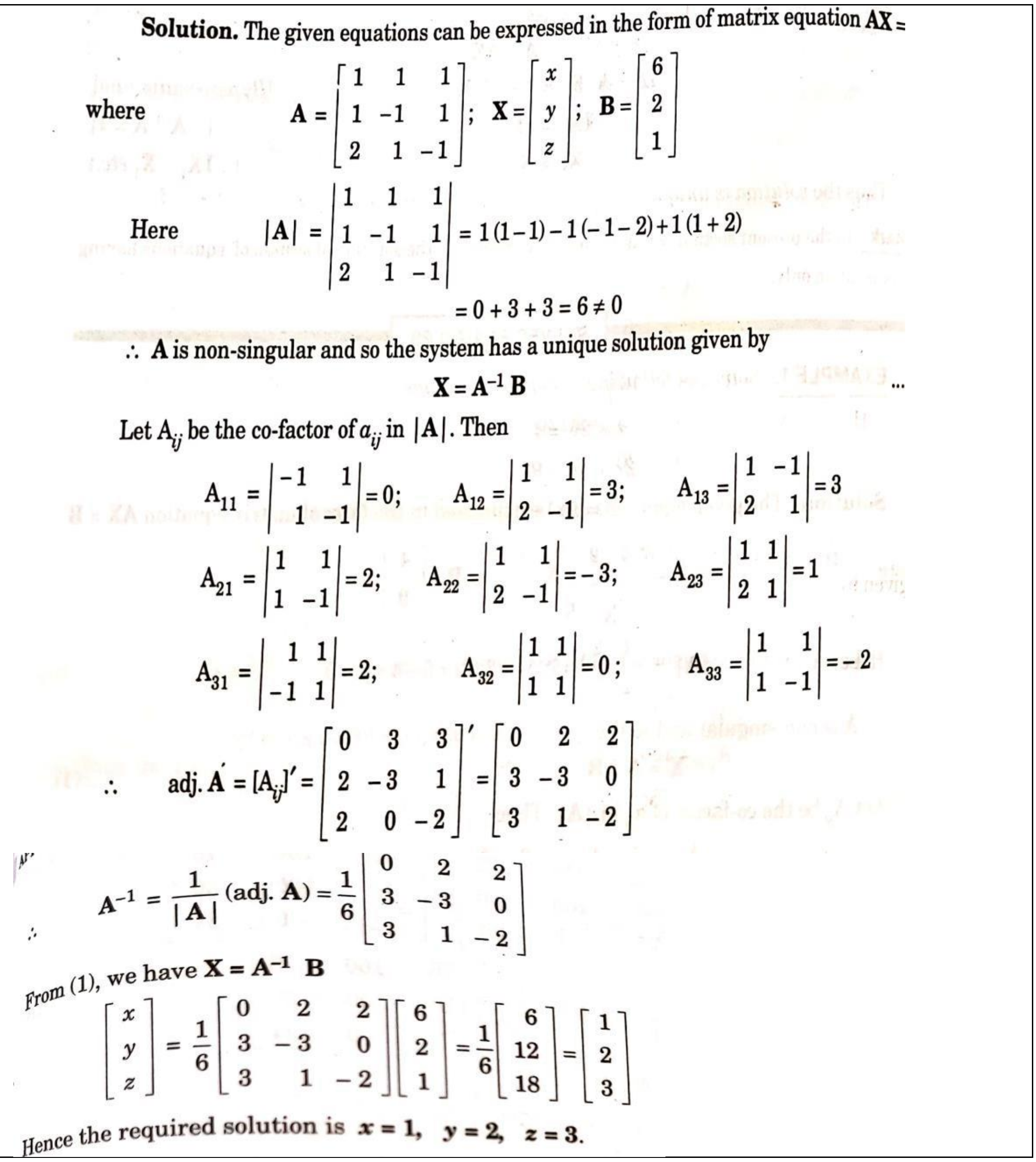


Solution. The given equations can be expressed in the form of matrix equation **AX** =

where $$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}; \quad \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

Here $$|\mathbf{A}| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1(1-1) - 1(-1-2) + 1(1+2)$$

$$= 0 + 3 + 3 = 6 \neq 0$$

$\therefore$ **A** is non-singular and so the system has a unique solution given by

$$\mathbf{X} = \mathbf{A}^{-1}\,\mathbf{B}$$

Let A_{ij} be the co-factor of a_{ij} in $|\mathbf{A}|$. Then

$$A_{11} = \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = 0; \quad A_{12} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = 3; \quad A_{13} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 3$$

$$A_{21} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 2; \quad A_{22} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3; \quad A_{23} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1$$

$$A_{31} = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2; \quad A_{32} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0; \quad A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2$$

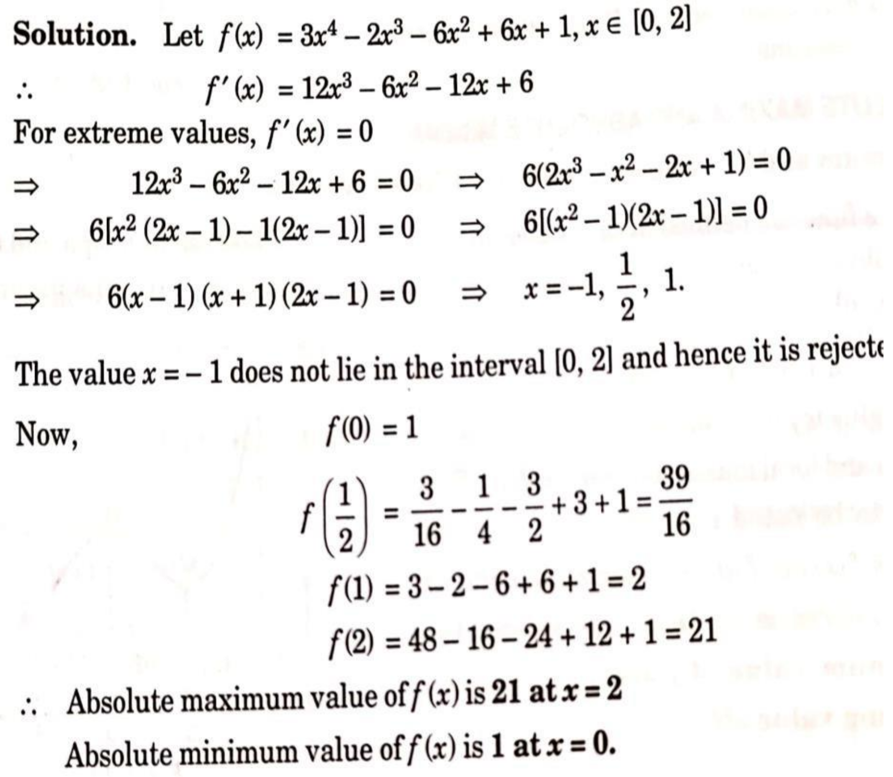
$$\therefore \quad \text{adj. } \mathbf{A}' = [A_{ij}]' = \begin{bmatrix} 0 & 3 & 3 \\ 2 & -3 & 1 \\ 2 & 0 & -2 \end{bmatrix}' = \begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}$$

$$\therefore \quad \mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|}(\text{adj. } \mathbf{A}) = \frac{1}{6}\begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}$$

From (1), we have $\mathbf{X} = \mathbf{A}^{-1}\ \mathbf{B}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence the required solution is $\mathbf{x = 1}$, $\mathbf{y = 2}$, $\mathbf{z = 3}$.

33	 **Solution.** Let $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1, x \in [0, 2]$ $\therefore \quad f'(x) = 12x^3 - 6x^2 - 12x + 6$ For extreme values, $f'(x) = 0$ $\Rightarrow \quad 12x^3 - 6x^2 - 12x + 6 = 0 \quad \Rightarrow \quad 6(2x^3 - x^2 - 2x + 1) = 0$ $\Rightarrow \quad 6[x^2(2x-1) - 1(2x-1)] = 0 \quad \Rightarrow \quad 6[(x^2-1)(2x-1)] = 0$ $\Rightarrow \quad 6(x-1)(x+1)(2x-1) = 0 \quad \Rightarrow \quad x = -1, \frac{1}{2}, 1.$ The value $x = -1$ does not lie in the interval $[0, 2]$ and hence it is rejecte Now, $f(0) = 1$ $f\left(\frac{1}{2}\right) = \frac{3}{16} - \frac{1}{4} - \frac{3}{2} + 3 + 1 = \frac{39}{16}$ $f(1) = 3 - 2 - 6 + 6 + 1 = 2$ $f(2) = 48 - 16 - 24 + 12 + 1 = 21$ $\therefore$ Absolute maximum value of $f(x)$ is **21 at $x = 2$** Absolute minimum value of $f(x)$ is **1 at $x = 0$.** OR

Solution. Here $f(x) = (x-1)^3 (x-2)^2$

$\therefore \quad f'(x) = (x-1)^3 . 2(x-2) + (x-2)^2 . 3(x-1)^2$

$= (x-1)^2 (x-2) [2(x-1) + 3(x-2)] = (x-1)^2 (x-2) (5x-8)$

Now, $f'(x) = 0 \Rightarrow (x-1)^2 (x-2) (5x-8) = 0$

$\Rightarrow x = 1, 2, \frac{8}{5}$, which are the critical values

These values of x give rise to following intervals :

(*a*) $x < 1$ (*b*) $1 < x < \frac{8}{5}$ (*c*) $\frac{8}{5} < x < 2$ (*d*) $x > 2$

When $x < 1$, $f'(x) = (+\text{ ve}) (-\text{ ve}) (-\text{ ve}) = +\text{ ve}$

$\therefore$ $f(x)$ is strictly increasing for $x < 1$.

When $1 < x < \frac{8}{5}$, $f'(x) = (+\text{ ve}) (-\text{ ve}) (-\text{ ve}) = +\text{ ve}$

$\therefore$ $f(x)$ is strictly increasing for $1 < x < \frac{8}{5}$

When $\frac{8}{5} < x < 2$, $f'(x) = (+\text{ ve}) (-\text{ ve}) (+\text{ ve}) = -\text{ ve}$

$\therefore$ $f(x)$ is strictly decreasing for $\frac{8}{5} < x < 2$

When $x > 2$, $f'(x) = (+\text{ ve}) (+\text{ ve}) (+\text{ ve}) = +\text{ ve}$

$\therefore$ $f(x)$ is strictly increasing for $x > 2$

Hence, $f(x)$ is strictly increasing for $x < 1$, $1 < x < \frac{8}{5}$ and $x > 2$

i.e., on $(-\infty, 1) \cup \left(1, \frac{8}{5}\right) \cup (2, \infty)$ and strictly decreasing for $\frac{8}{5} < x < 2$ *i.e.,* on $\left(\frac{8}{5}, 2\right)$

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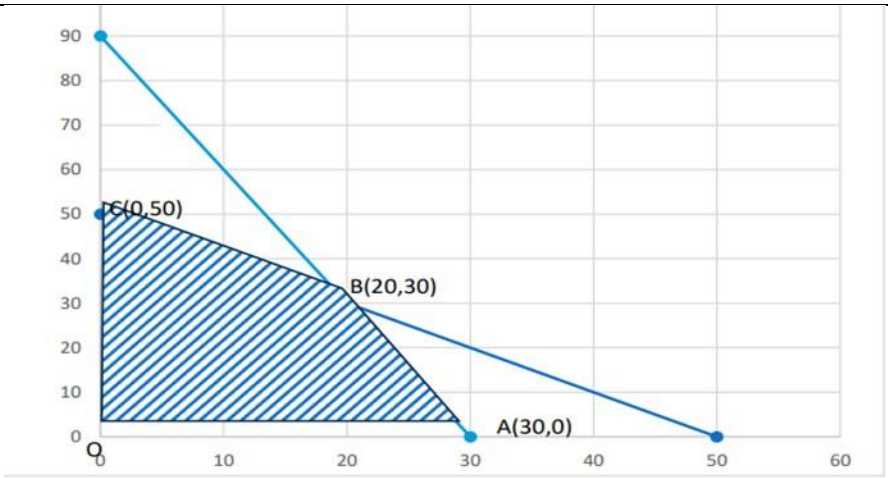

Corner point	Z=4X+Y
O(0,0)	0
A(30,0)	120
B(20,30)	110
C(0,50	50

As Feasible region bounded (so by corner point theorem)

Hence, maximum value of Z is 120 at point (30,0)

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When a die is tossed S={1.2.3.4.5.6}

Let E be the event ''the number greater than 4''

E={5.6}

n (E)=2

p=P(E) $=\frac{2}{6}=\frac{1}{3}$

q=1- $\frac{1}{3}=\frac{2}{3}$

As the die is tossed twice there are 2 bernoullian trials.

Let X denote the number of successes the X can take values 0,1,2

P(0) = $2C_0\, p^0q^2 = (\frac{2}{3})^2 = \frac{4}{9}$

P(1) = $2C_1\, p^1q^1 = 2\left(\frac{1}{3}\right)(\frac{2}{3}) = \frac{4}{9}$

P(2) = $2C\, p^2q^0 = (\frac{1}{3})^2 = \frac{1}{9}$

Probability distribution of Number of successes is

x	0	1	2
P(x)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

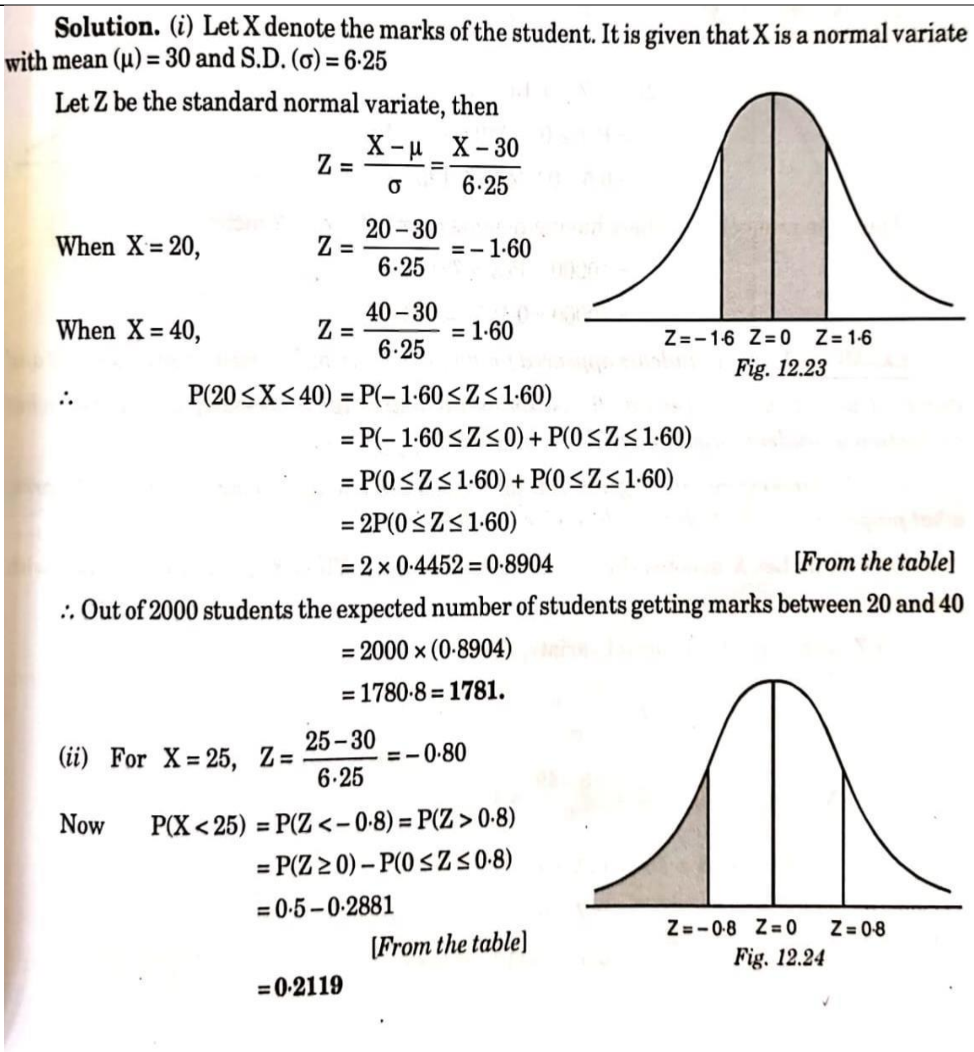
Mean $=\Sigma x_i p_i = (0)(\frac{4}{9}) + (1)\left(\frac{4}{9}\right) + 2\left(\frac{1}{9}\right) = \frac{6}{9}$

$\Sigma p_i x^2 = (0)(\frac{4}{9}) + (1)\left(\frac{4}{9}\right) + 4\left(\frac{1}{9}\right) = \frac{8}{9}$

Variance= $\Sigma p_i x^2 - (\Sigma x_i p_i)^2 = \frac{8}{9} - \frac{4}{9} = \frac{4}{9}$

Standard Deviation= $\sqrt{Var} = \frac{2}{3}$.

OR

	 **Solution.** (*i*) Let X denote the marks of the student. It is given that X is a normal variate with mean (μ) = 30 and S.D. (σ) = 6·25 Let Z be the standard normal variate, then $Z = \frac{X-\mu}{\sigma} = \frac{X-30}{6\cdot 25}$ When X = 20, $Z = \frac{20-30}{6\cdot 25} = -1\cdot 60$ When X = 40, $Z = \frac{40-30}{6\cdot 25} = 1\cdot 60$ $\therefore$ $P(20 \le X \le 40) = P(-1\cdot 60 \le Z \le 1\cdot 60)$ $= P(-1\cdot 60 \le Z \le 0) + P(0 \le Z \le 1\cdot 60)$ $= P(0 \le Z \le 1\cdot 60) + P(0 \le Z \le 1\cdot 60)$ $= 2P(0 \le Z \le 1\cdot 60)$ $= 2 \times 0\cdot 4452 = 0\cdot 8904$ [*From the table*] *Fig. 12.23* $\therefore$ Out of 2000 students the expected number of students getting marks between 20 and 40 $= 2000 \times (0\cdot 8904)$ $= 1780\cdot 8 =$ **1781.** (*ii*) For X = 25, $Z = \frac{25-30}{6\cdot 25} = -0\cdot 80$ Now $P(X < 25) = P(Z < -0\cdot 8) = P(Z > 0\cdot 8)$ $= P(Z \ge 0) - P(0 \le Z \le 0\cdot 8)$ $= 0\cdot 5 - 0\cdot 2881$ [*From the table*] $=$ **0·2119** *Fig. 12.24*
36	(i) Pipe C empties 2/5th of tank in 2/5*20 =8hrs (ii) Part of tank filled in 1 hr =1/15+1/12-1/20 = 1/10 Time = 10hrs (iii) Part filled by A and B = t/15 and (t-3)/15 Emptied by C = (t-4)/20 t/15+(t-3)/15- (t-4)/20 =1 t=10.5 hrs OR Part filled by A and B in 1 hr = 1/15+1/20 = 3/20 Time = 20/3 hrs Total time = 10+1+20/3 = 17 hr 40 mins

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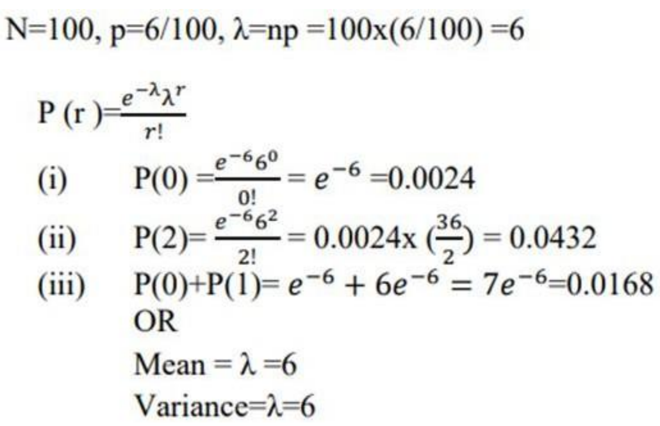

N=100, p=6/100, λ=np =100x(6/100) =6

$P(r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$

(i) $P(0) = \frac{e^{-6}6^{0}}{0!} = e^{-6} = 0.0024$

(ii) $P(2) = \frac{e^{-6}6^{2}}{2!} = 0.0024x\ (\frac{36}{2}) = 0.0432$

(iii) $P(0)+P(1) = e^{-6} + 6e^{-6} = 7e^{-6} = 0.0168$

OR

Mean = λ =6

Variance=λ=6

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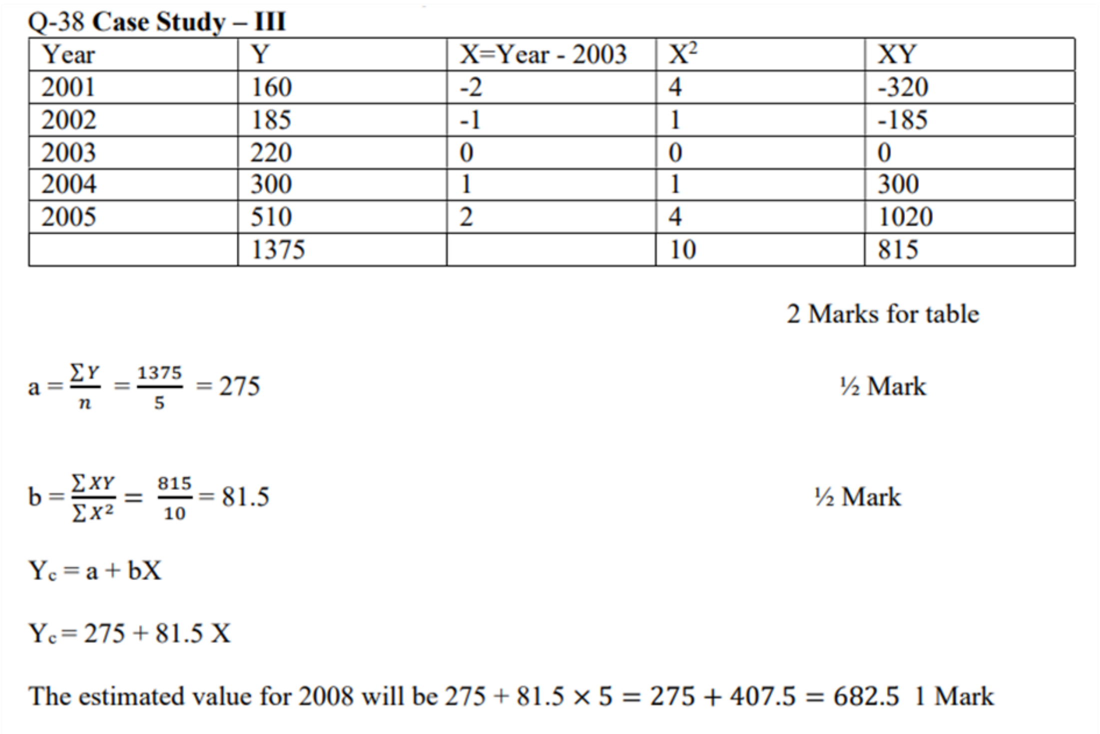

Q-38 **Case Study – III**

Year	Y	X=Year - 2003	X^2	XY
2001	160	-2	4	-320
2002	185	-1	1	-185
2003	220	0	0	0
2004	300	1	1	300
2005	510	2	4	1020
	1375		10	815

2 Marks for table

$a = \frac{\sum Y}{n} = \frac{1375}{5} = 275$ ½ Mark

$b = \frac{\sum XY}{\sum X^2} = \frac{815}{10} = 81.5$ ½ Mark

$Y_c = a + bX$

$Y_c = 275 + 81.5\ X$

The estimated value for 2008 will be $275 + 81.5 \times 5 = 275 + 407.5 = 682.5$ 1 Mark

*****END*****